# Ground-based detection of gravitational waves from intermediate-mass-ratio inspirals

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based on
Mandel, Brown, Gair, Miller: ApJ 681 1431 (2008)
Gair, Li, Mandel: PRD 77 024035 (2008)
Mandel: arXiv:0707.0711

# Outline

- GWs in ground-based detectors
- Intermediate-mass-ratio inspirals into intermediate-mass black holes: rates and characteristics
- Probing the strong field region near a black hole

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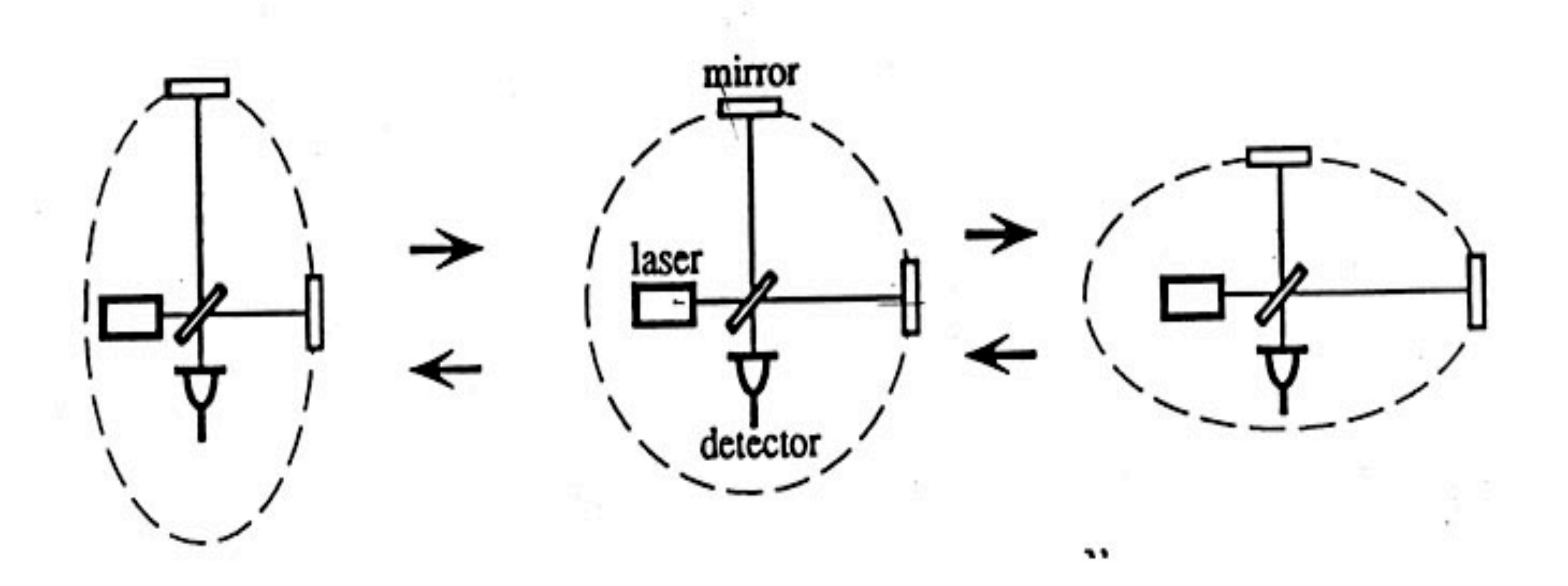
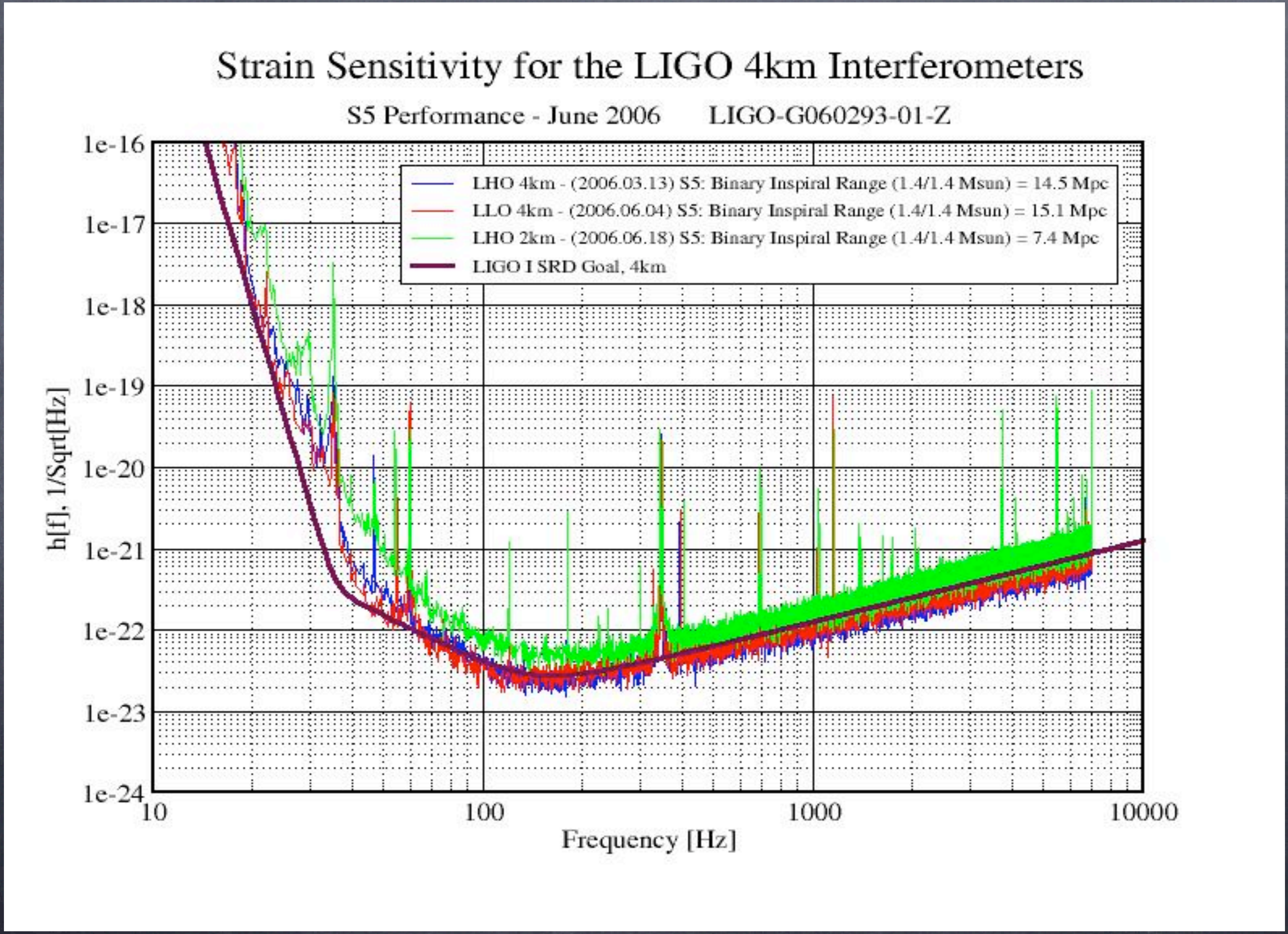
# GW detection with Michelson-type interferometers



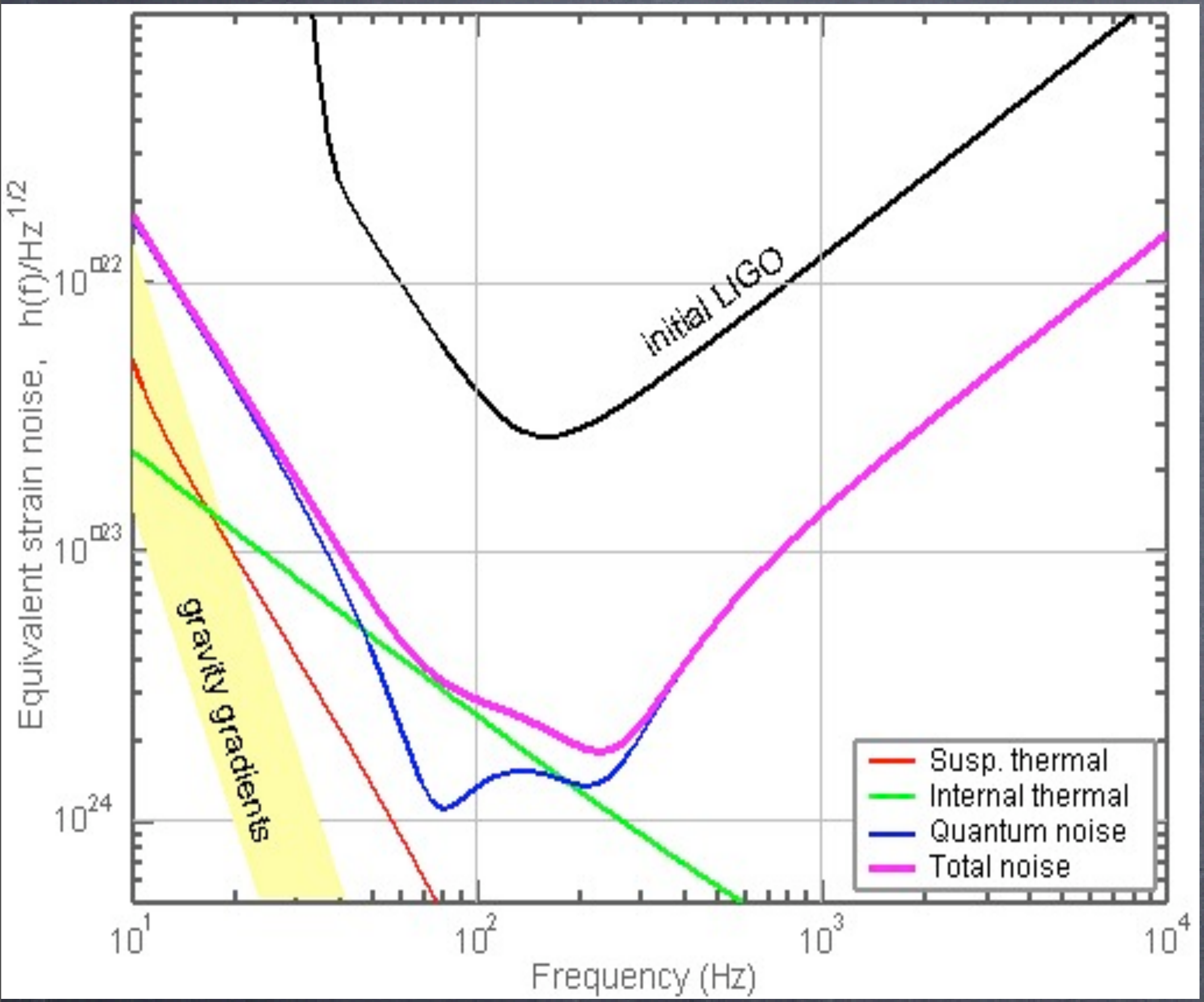
# LIGO

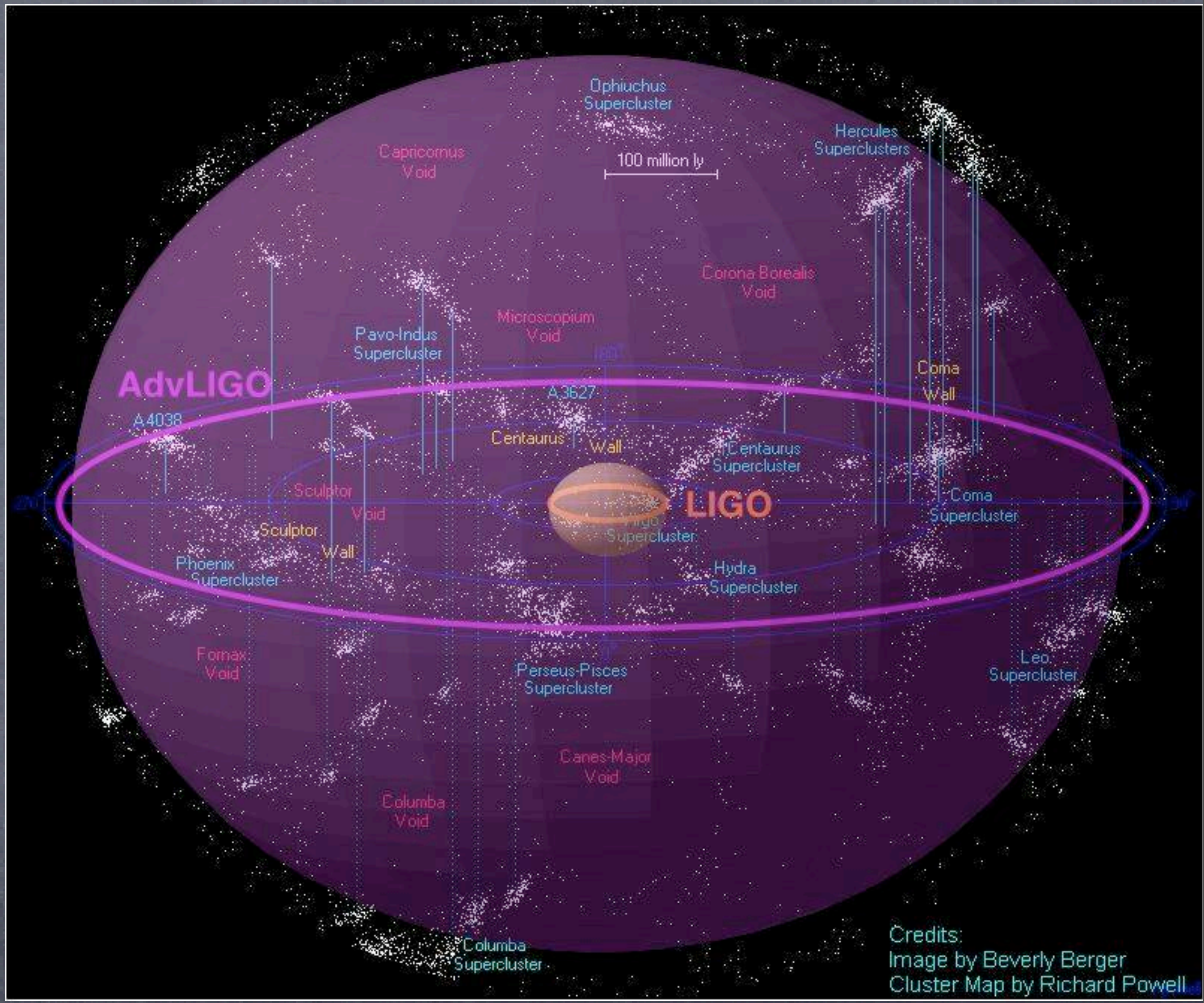
- 4 km long arms
- Typical strains $h = \Delta L / L \sim 10^{-22}$
- Needs to measure $\Delta L = hL \sim 10^{-17}$ m

# LIGO Noise Curve



# Advanced LIGO




- x10 in range -> x1000 in event rate
- 10 Hz low frequency cutoff

# Outline

- GWs in ground-based detectors
- Intermediate-mass-ratio inspirals into intermediate-mass black holes: rates and characteristics
- Probing the strong field region near a black hole: testing the no-hair theorem

# Intermediate-mass-ratio inspirals (IMRIs)

- IMRIs have mass ratios between 10 and $10^4$
- LIGO IMRIs: Inspirals of compact objects (1.4 solar-mass Neutrons Stars to 10 solar-mass Black Holes) into intermediate mass black holes (IMBHs, 50-350 solar masses)
- Indirect evidence for IMBH existence in globular clusters (50 – $10^4$ solar masses)
  - Observational evidence (e.g. Macarone et al.)
  - Simulations (e.g. McMillan et al., O'Leary et al.)
  - Simulations vs. Observations (e.g. Trenti)
- IMRIs could be the first proof of IMBH existence!

# Event Rates: Mechanisms

- Three-body interactions: IMBH swaps into binaries, forms CO-IMBH binaries which are tightened via three-body interactions with other stars, then merge via GW radiation reaction
- Direct capture via energy loss to GWs
- Kozai resonances in hierarchical triple systems: inner binary eccentricity is driven up by outer companion
- Tidal capture of MS star that evolves into CO while in orbit
- Tidal interactions (orbital-vibrational coupling) for NS inspirals

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# Event rates per G.C.

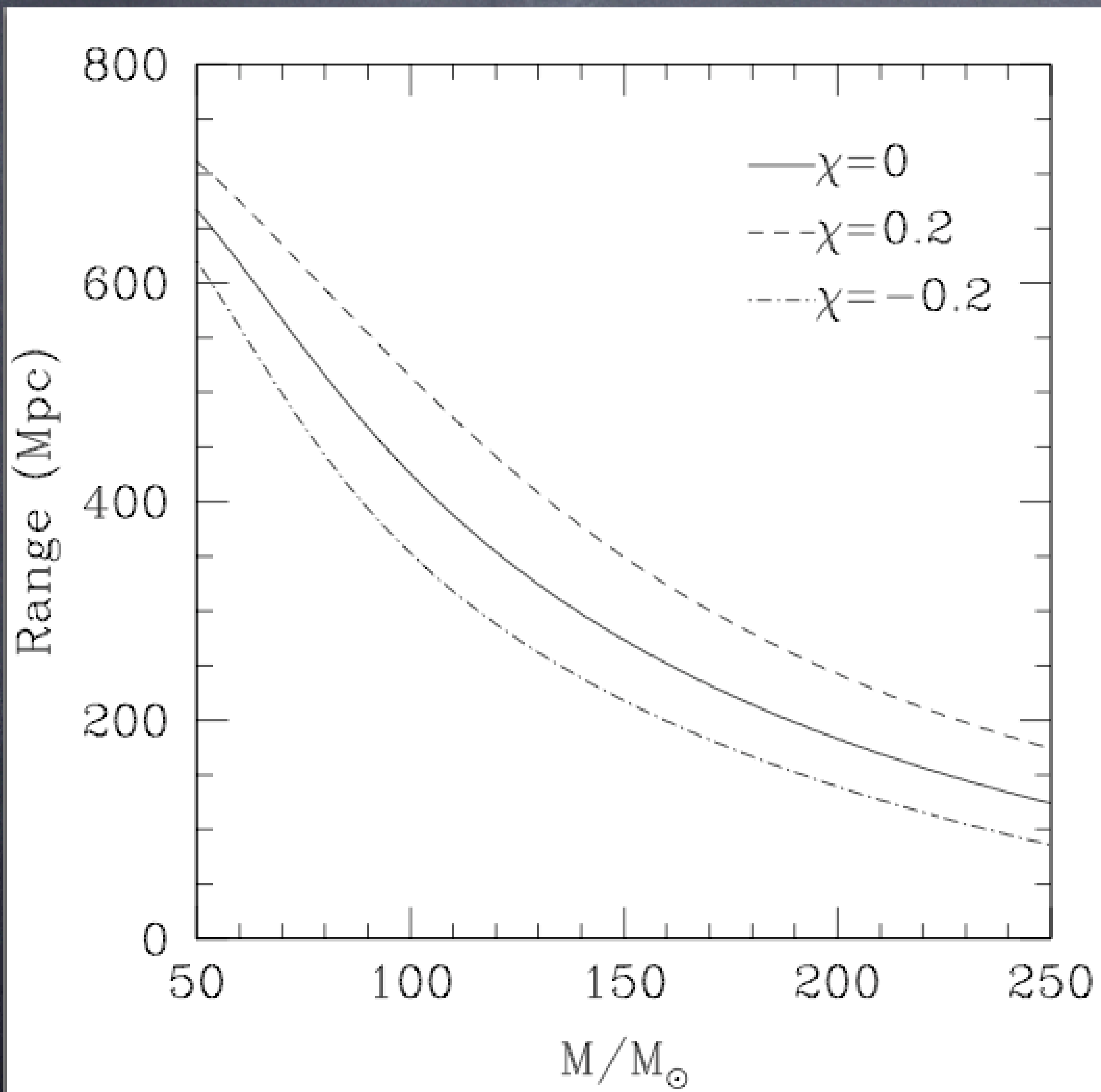
- Binary tightening via 3-body interaction
- 3-body interaction rate is $dN/dt = n\sigma v$;

  $n \sim 10^{5.5}$ pc$^{-3}$; $v \sim 10$ km/s; $\sigma \sim \pi a(2GM/v^2)$
- $T_{harden} \sim O(M/m)\,(dN/dt)^{-1} \sim 1.5 * 10^8\,(AU/a)$ yr [Quinlan]
- $T_{merge} \sim 5 * 10^{17}\, M_\bullet^3/(M^2 m)\,(a/AU)^4\,(1-e^2)^{7/2}$ yr

  $\sim 5 * 10^8\,(M_\bullet/m)\,(100 M_\bullet/M)^2\,(a/AU)^4$ yr [Peters & Mathews]
- To maximize rate, minimize $T = T_{harden} + T_{merge}$
- Rate per globular is $\sim 3 * 10^{-9}$ yr$^{-1}$ for NS,

  $5 * 10^{-9}$ yr$^{-1}$ for BH

# Advanced LIGO IMRI sensitivity




- Use EMRI-like waveforms, including non-quadrupolar harmonics, to determine range
- Range is spin-dependent

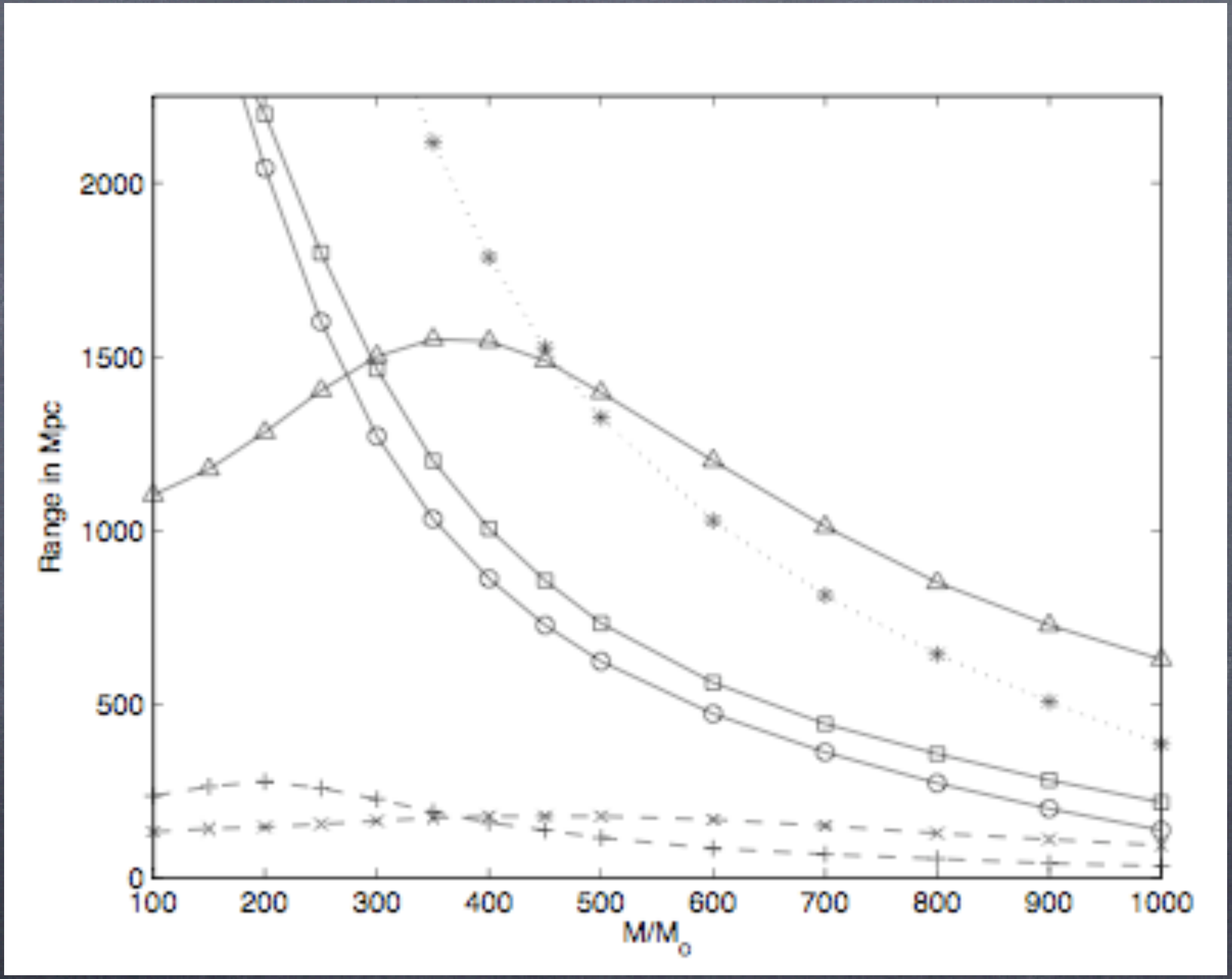
$$R \approx \left[1 + (\chi^2/2)\left(\frac{M}{100\,M_\odot}\right)^{1.5}\right]\sqrt{\frac{m}{M_\odot}}\left[800 - 540\left(\frac{M}{100\,M_\odot}\right) + 107\left(\frac{M}{100\,M_\odot}\right)^2\right]$$

- Range could be increased by x1.5 by tuning Advanced LIGO

# Advanced LIGO IMRI rates

- Assume 10% of all globular clusters hold suitable IMBH (typical mass 100 Msun, spin=0.2)
- If inspiraling object is 1.4 Msun NS, Advanced LIGO could detect one IMRI per 3 years
- If inspiraling object is 10 Msun BH, Advanced LIGO could detect 10 IMRIs per year
- If Advanced LIGO is IMRI-optimized, rates could go up to 1/year and 30/year

# Ringdowns




Could complement IMRIs if higher CO and IMBH masses are prevalent

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idea taken from Daniel Shaddock

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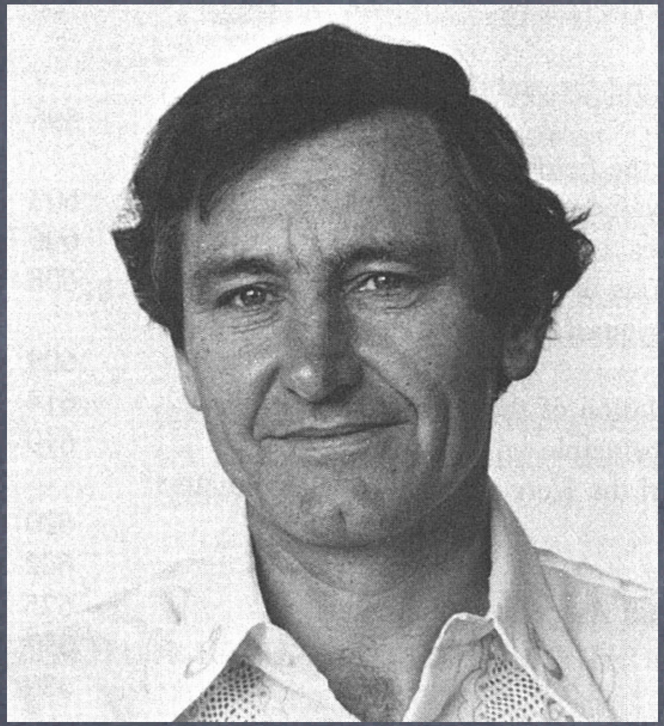
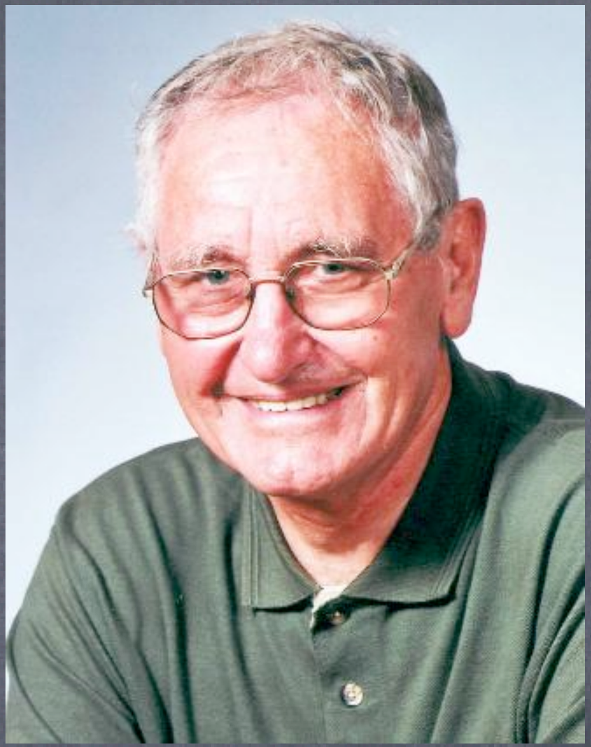




Stationary, vacuum, asymptotically flat spacetimes in which the singularity is fully enclosed by a horizon with no closed timelike curves outside the horizon are described by the Kerr metric

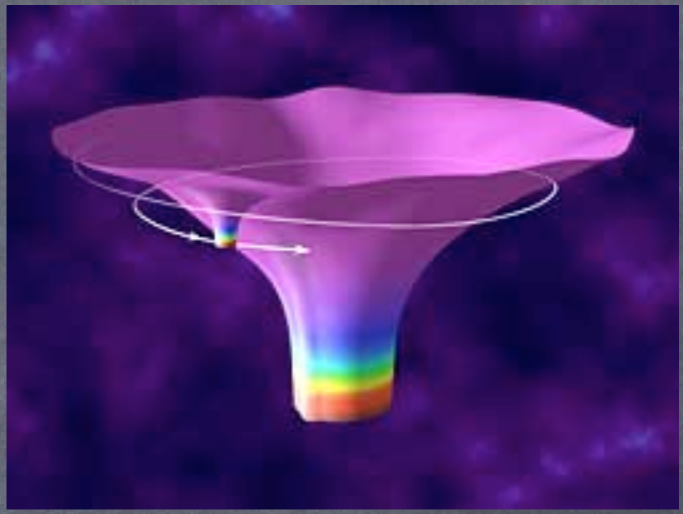
# The no-hair theorem in English

- "Black holes have no hair" means that all higher-order mass and current multipole moments are uniquely determined by the black hole mass and spin
- Conversely, an object with hair is one for which $$M_n + iS_n \neq M(ia)^n$$
- The "no-hair theorem" is a mathematical statement, so the title is a bit of a misnomer...

# New subtitle: do massive black holes have hair?

- Are massive "black holes" really black holes?
- Could they be boson stars, or naked singularities, or...?
- Need to measure 3 multipole moments to test "Kerrness", 4 to test if an object is a boson star
- Search for exotic massive compact objects, test of cosmic censorship conjecture, null hypothesis test of the no-hair theorem...

# Do Black Holes Have Hair? Probing spacetime with E/IMRIs



- Are massive "black holes" really hairless?
- Or could they be boson stars, naked singularities, ...?
- Need to measure 3 multipole moments to test "Kerrness", 4 to test if an object is a boson star
- LISA EMRIs into SMBHs will be the best probes of the strong-field regime (#cycles ~ M/m), but Advanced LIGO IMRIs into IMBHs may provide the first interesting test
- Information about the spacetime structure and the orbit should be contained in GWs; how do we access it?

# Summary

- Advanced LIGO could detect a few IMRIs per year
- Eccentricities will be low, circular waveforms can be used for detection (But should we use EMRI waveforms? Hybrid waveforms? ...?)
- Gravitational waves from EMRIs should make it possible to test whether the central body [SMBH] is a Kerr black hole
- Chaos in a non-Kerr spacetime would be an obvious smoking gun, but chaotic regions are probably not accessible
- Location of ISCO, periapsis precession, and orbital-plane precession are possible observables indicating bumpiness
- Frequency evolution over inspiral would be another observable, but more work is required

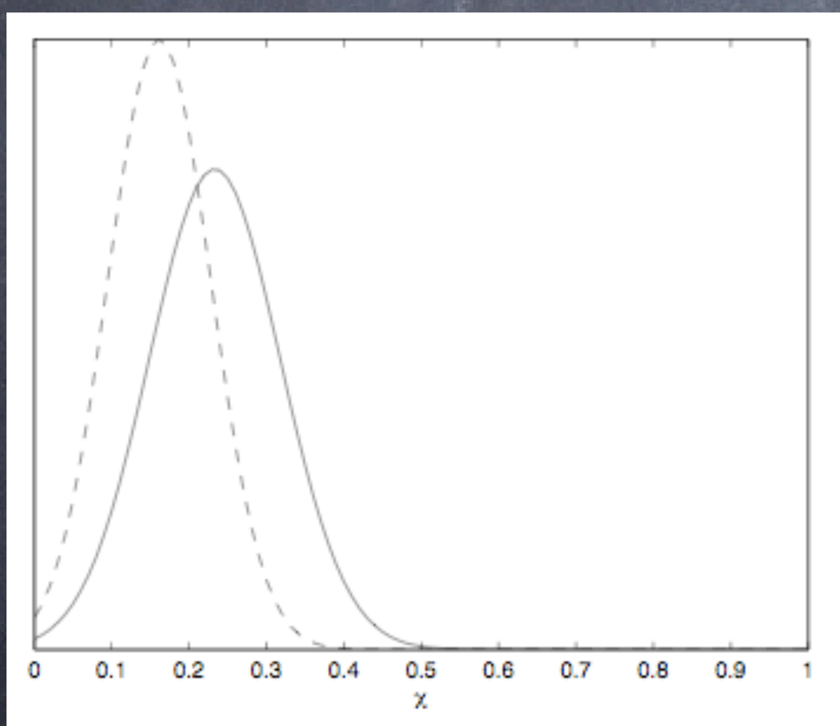
# Event rates – upper limit

- Model-independent upper limit
- One core-collapsed globular cluster per $Mpc^3$
- One suitable IMBH per globular cluster
- IMBH grows from 50 to 350 solar masses by capture of COs in Hubble time
- Advanced LIGO could see IMRIs up to ~1000 Mpc (depending on masses, spin)
- Advanced LIGO may see tens of IMRIs per year (only 1 in 1000 years with Initial LIGO)
- Issues: kicks above 50 km/s eject IMBH; lower rates late in cluster history [e.g. simulations by O'Leary et al. 2006]

# Spin and detection range

## Evolution of spin distribution via minor mergers

$$\frac{\partial}{\partial t} f(\chi,t) = -\frac{\partial}{\partial\chi}\left[\frac{\chi}{t}\left(-2-\frac{4\sqrt{2}}{9}+\frac{4}{\chi^2 t}\right) f(\chi,t)\right] + \frac{1}{2}\frac{\partial^2}{\partial\chi^2}\left[\frac{4}{t^2}\left(1+\frac{4\sqrt{2}\chi^2}{9}-\chi^2\right) f(\chi,t)\right]$$




t=M/m

$$\bar{\chi} \approx \bar{\chi}_0 \left(\frac{t}{t_0}\right)^{a} \approx \bar{\chi}_0 \left(\frac{M_0}{M}\right)^{2.63}$$

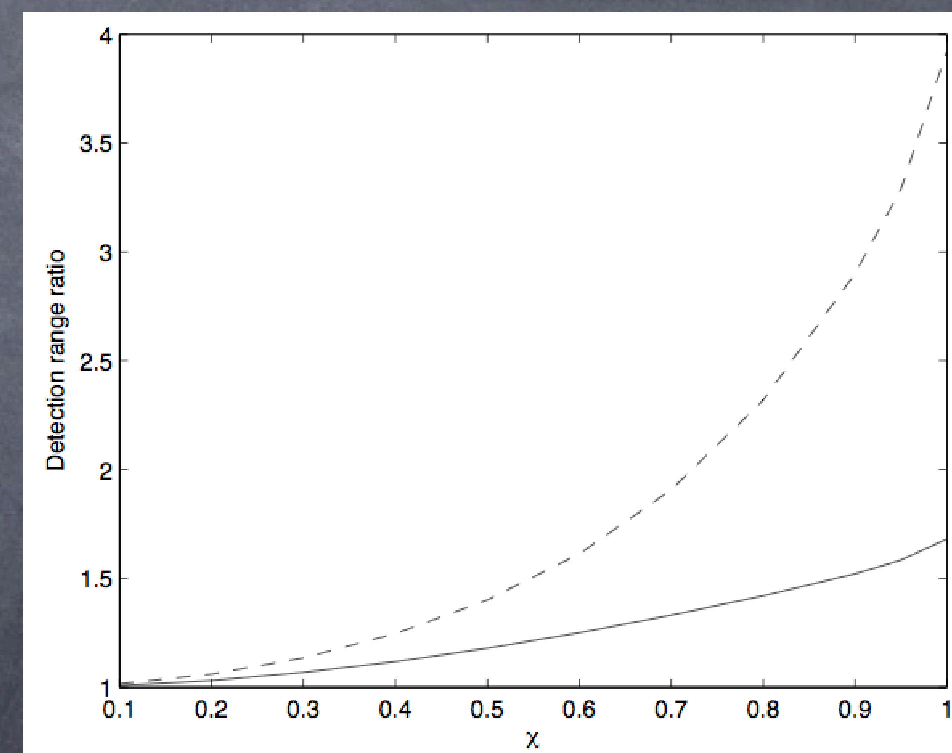
Evolution from t=M/m=50 to t=100 (e.g., from M=70 to M=140 solar masses via capture of m=1.4 solar-mass NSs)

If initial $\chi$=0.1, then mean spin at t=100 is 0.162, $\sigma$=0.066

If initial $\chi$=0.9, then mean spin at t=100 is 0.233, $\sigma$=0.087

## Range increase due to spin

$$\frac{\text{Range}_{\text{spin}}}{\text{Range}_{\text{no-spin}}} \sim 1 + 0.6\chi^2 \left(\frac{M}{100\ M_\odot}\right)$$



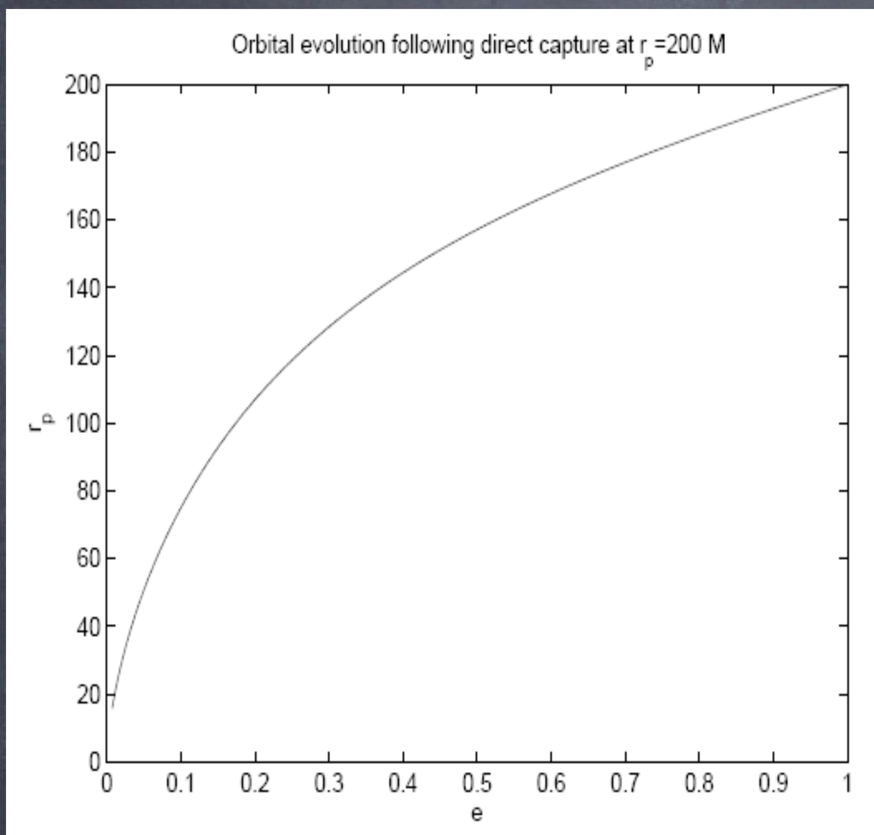

Solid line - inspiral into 100 Msun IMBH

Dashed line - inspiral into 200 Msun IMBH

Effect is very pronounced for LISA:

can cause bias in spin estimate

# Eccentricities in AdvLIGO band



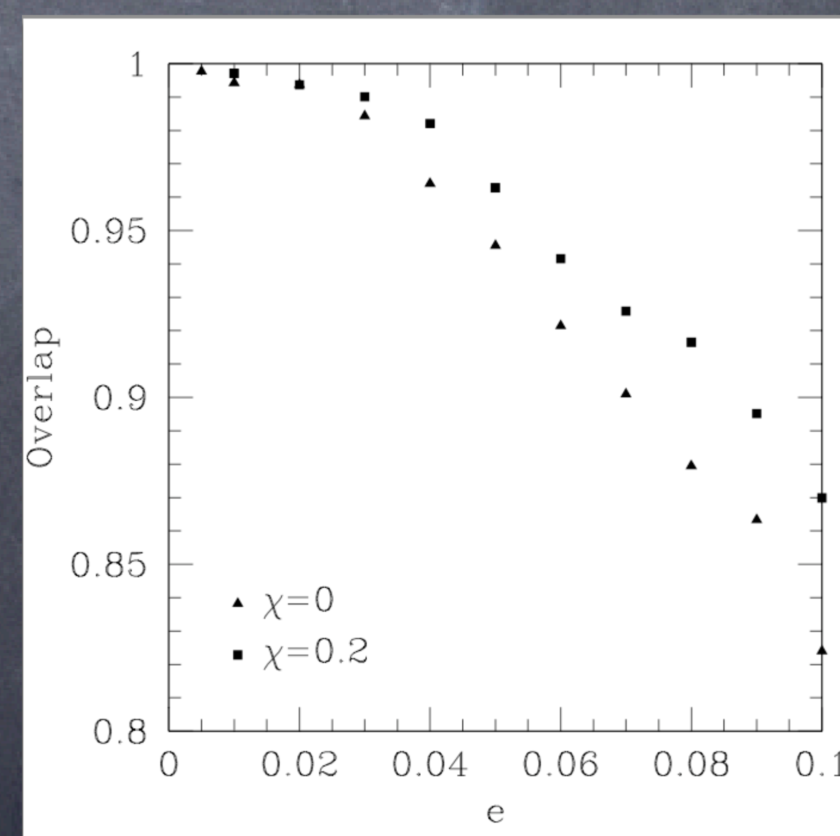

- Hardening via 3-body interactions

  Eccentricity ~ few*$10^{-5}$ when $f_{GW}$=10 Hz

- Direct capture

  90% of IMRIs circularize to e<0.1 by 10 Hz, 67% circularize to e<0.01 by $f_{GW}$=10 Hz

- At e=0.01, overlap between eccentric and circular templates is >0.99, so circular templates can be used for detection



# Observing deviations from Kerr with EMRIs

- LISA can detect tens to thousands of EMRIs
- Ryan's theorem [1995]: GWs from nearly circular, nearly equatorial orbits in stationary, axisymmetric spacetimes encode all of the spacetime multipole moments... in principle
- Can we extend this theorem? Are there obvious observable imprints of an anomalous, non-Kerr quadrupole moment (a "bumpy" spacetime)?
- Are energy E, angular momentum $L_z$ and Carter constant Q conserved in a bumpy spacetime?

# Geodesics in bumpy spacetimes

- Use Manko-Novikov bumpy spacetime

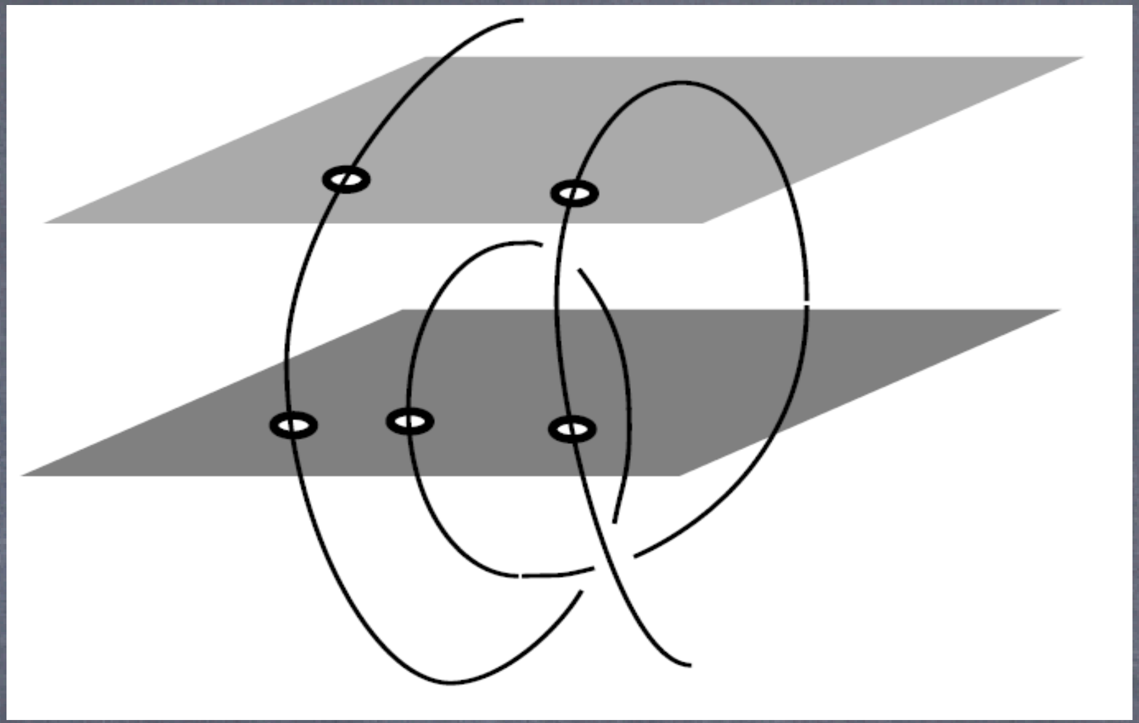
$$ds^2 = -f(\rho,z)\,(dt - \omega(\rho,z)\,d\phi)^2 + \frac{1}{f(\rho,z)}\left[e^{2\gamma(\rho,z)}\,(d\rho^2 + dz^2) + \rho^2\,d\phi^2\right]$$

- C code - geodesic equations:

$$\frac{\partial^2 x^\alpha}{\partial \tau^2} = -\Gamma^\alpha_{\beta\gamma}\frac{\partial x^\beta}{\partial \tau}\frac{\partial x^\gamma}{\partial \tau}$$

- Check conservation of E, $L_z$, 4-velocity norm
- Equations might not separate as in Kerr
- Is there a full set of integrals of motion?

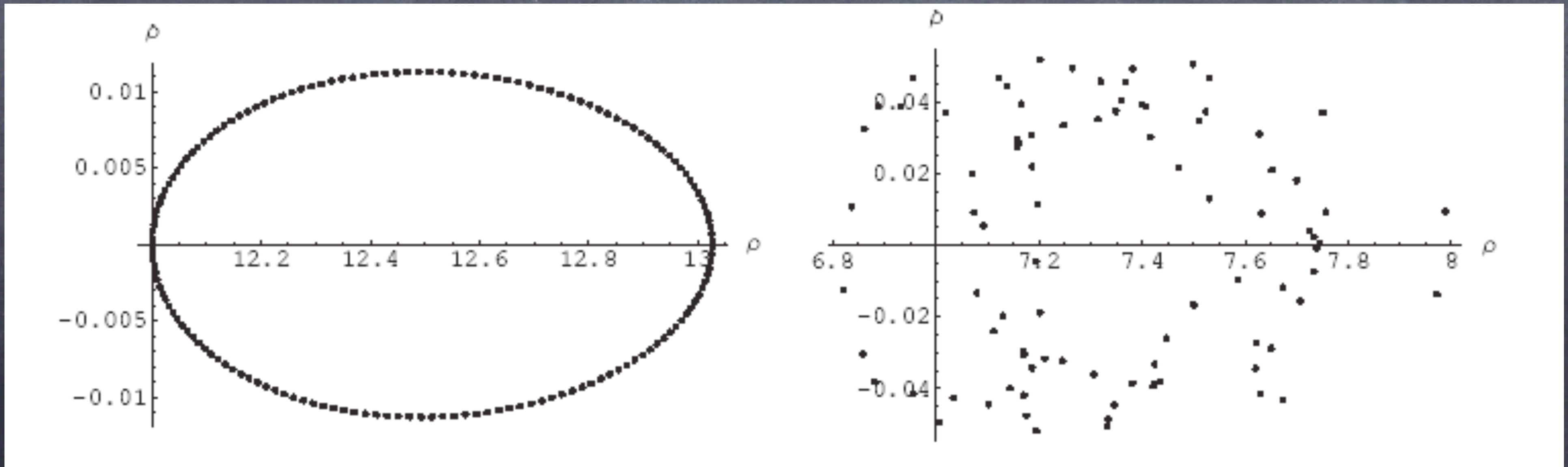
# Poincare maps



- Check if spacetime has a full set of integrals of motion
- Plot $d\rho/dt$ vs. $\rho$ for $z=z_0$ crossings
- Phase space plots should be closed curves for all $z_0$ iff there is a third isolating integral

# Poincare maps for motion in Newtonian potential with hexadecapole moment

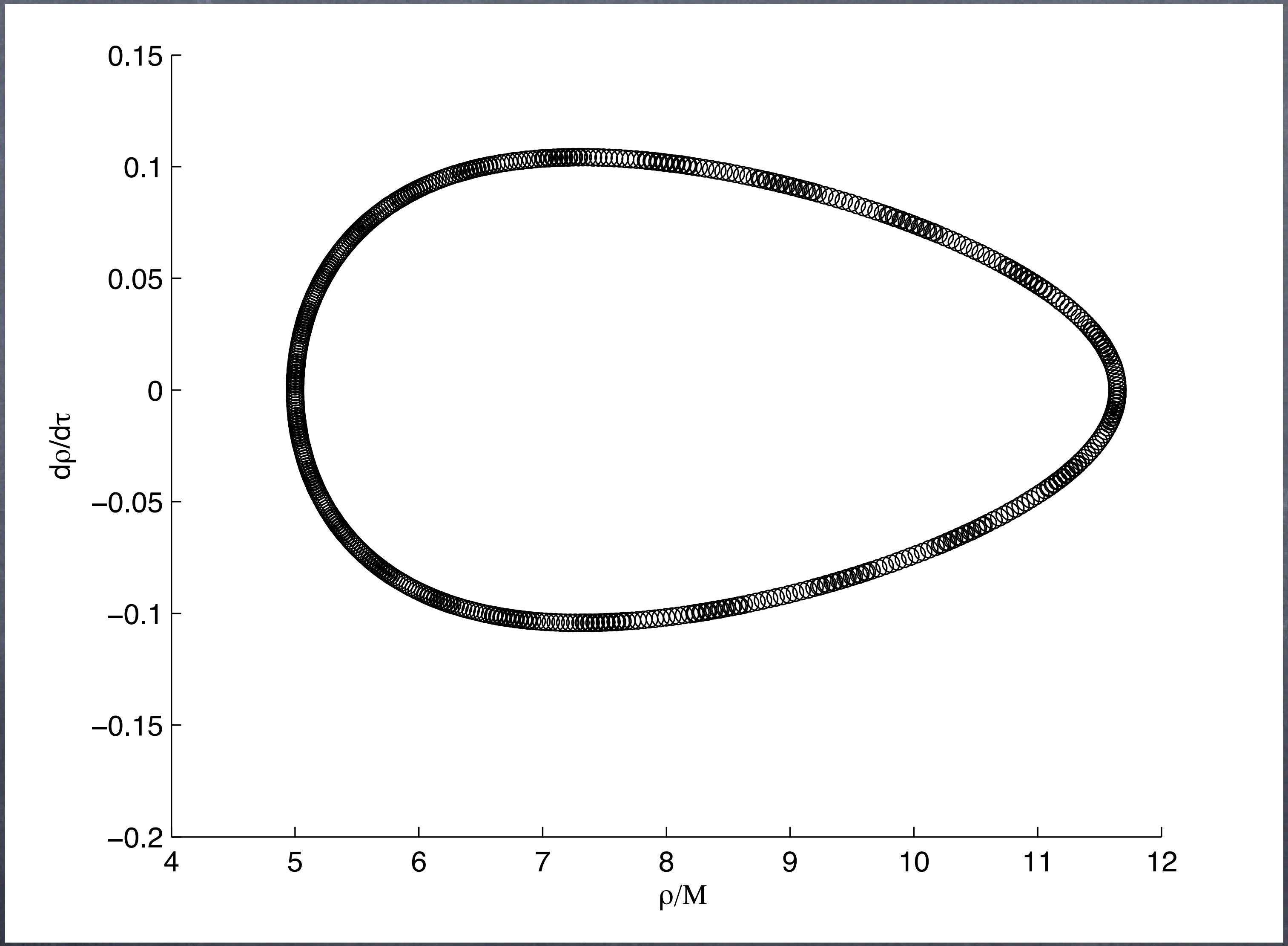
$$V(r,\theta) = -\frac{M_0}{r} + \frac{M_2}{r^3}P_2(\cos\theta) + \frac{M_4}{r^5}P_4(\cos\theta)$$




$M_2=10\ M_0$;  $M_4=400\ M_0$

Li

# Poincare map in a bumpy spacetime




E=0.95, $L_z$=-3, a/M=0.9, q=0.95

# Allowed regions for bound orbits

Effective potential $(\dot{\rho}^2 + \dot{z}^2) = V(E, L_z, \rho, z)$

z

ρ

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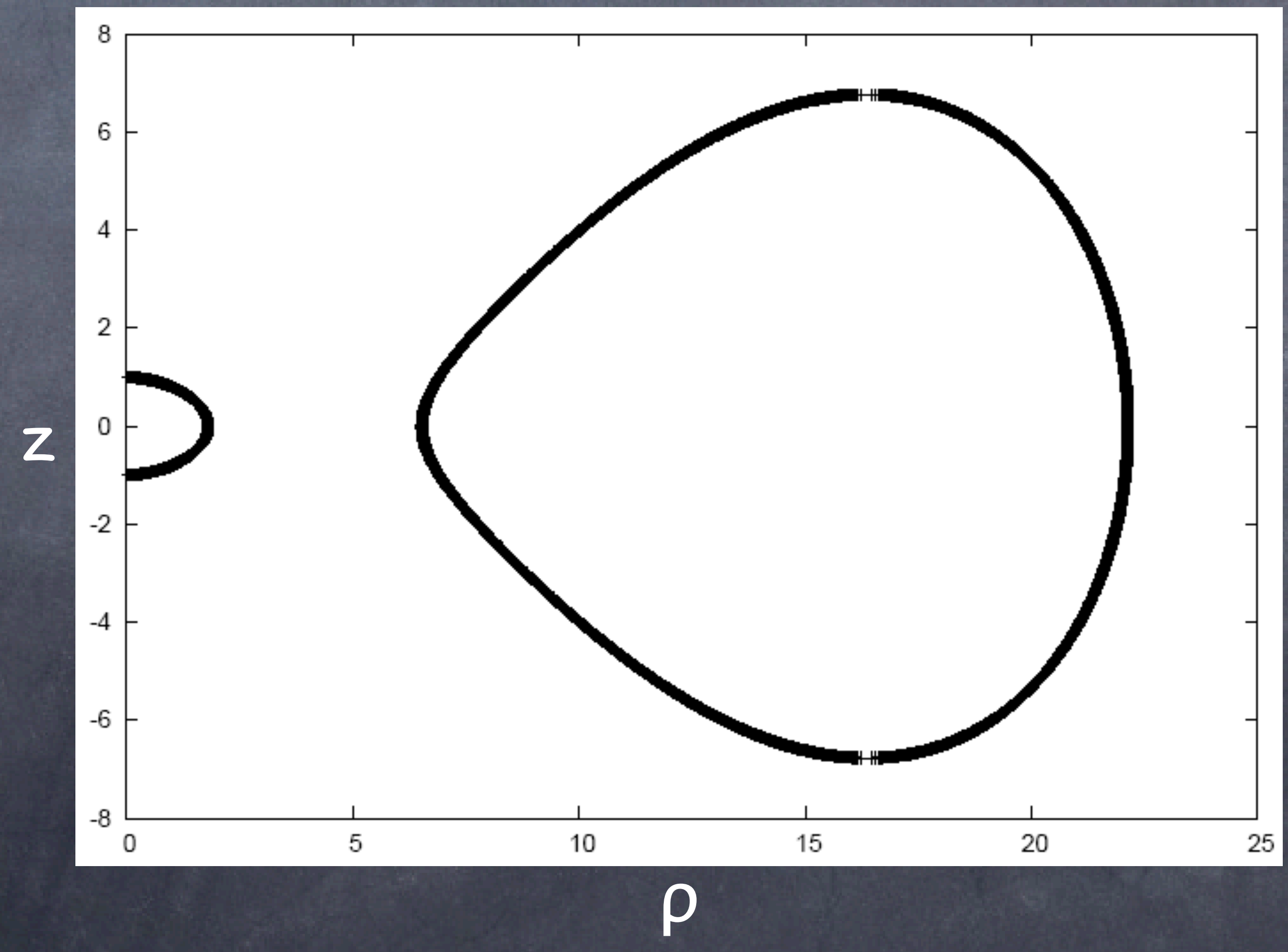


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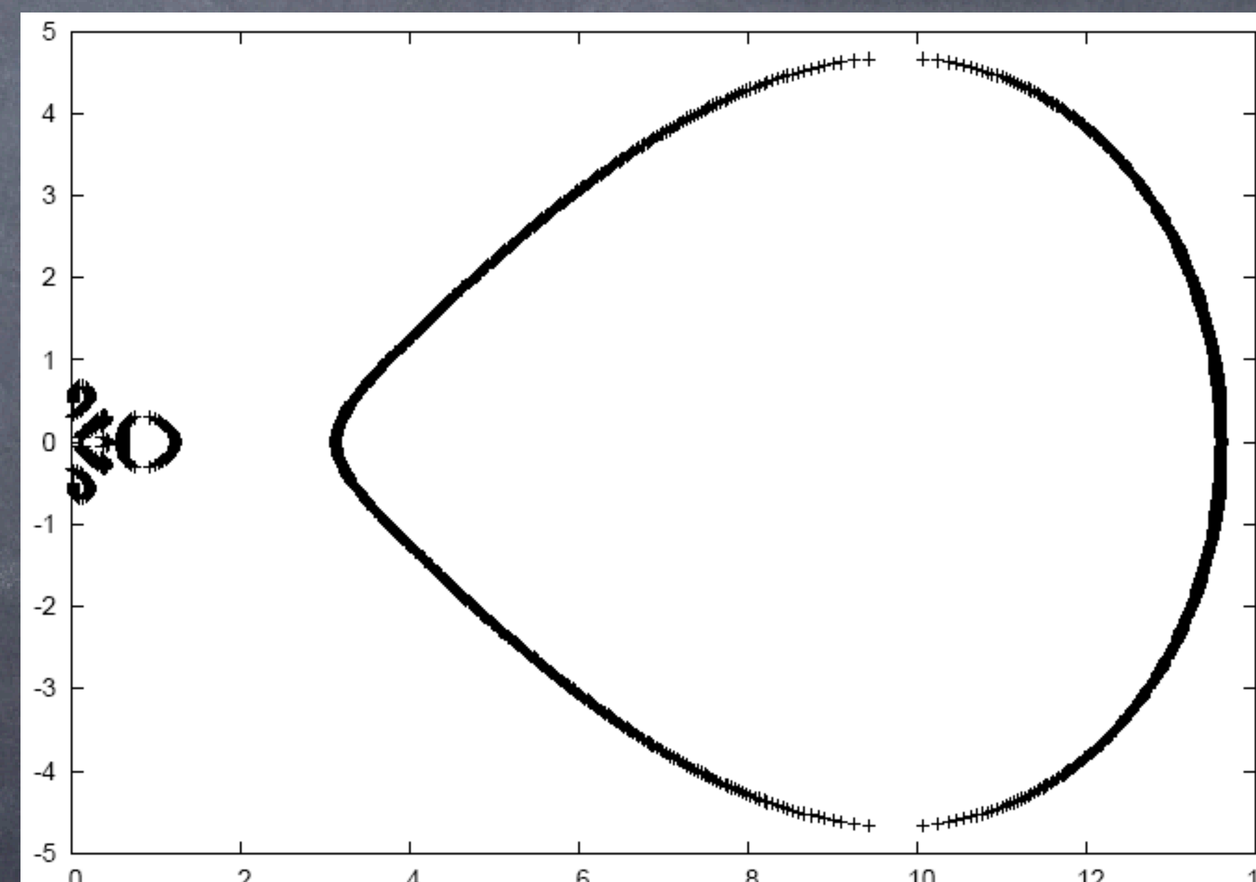


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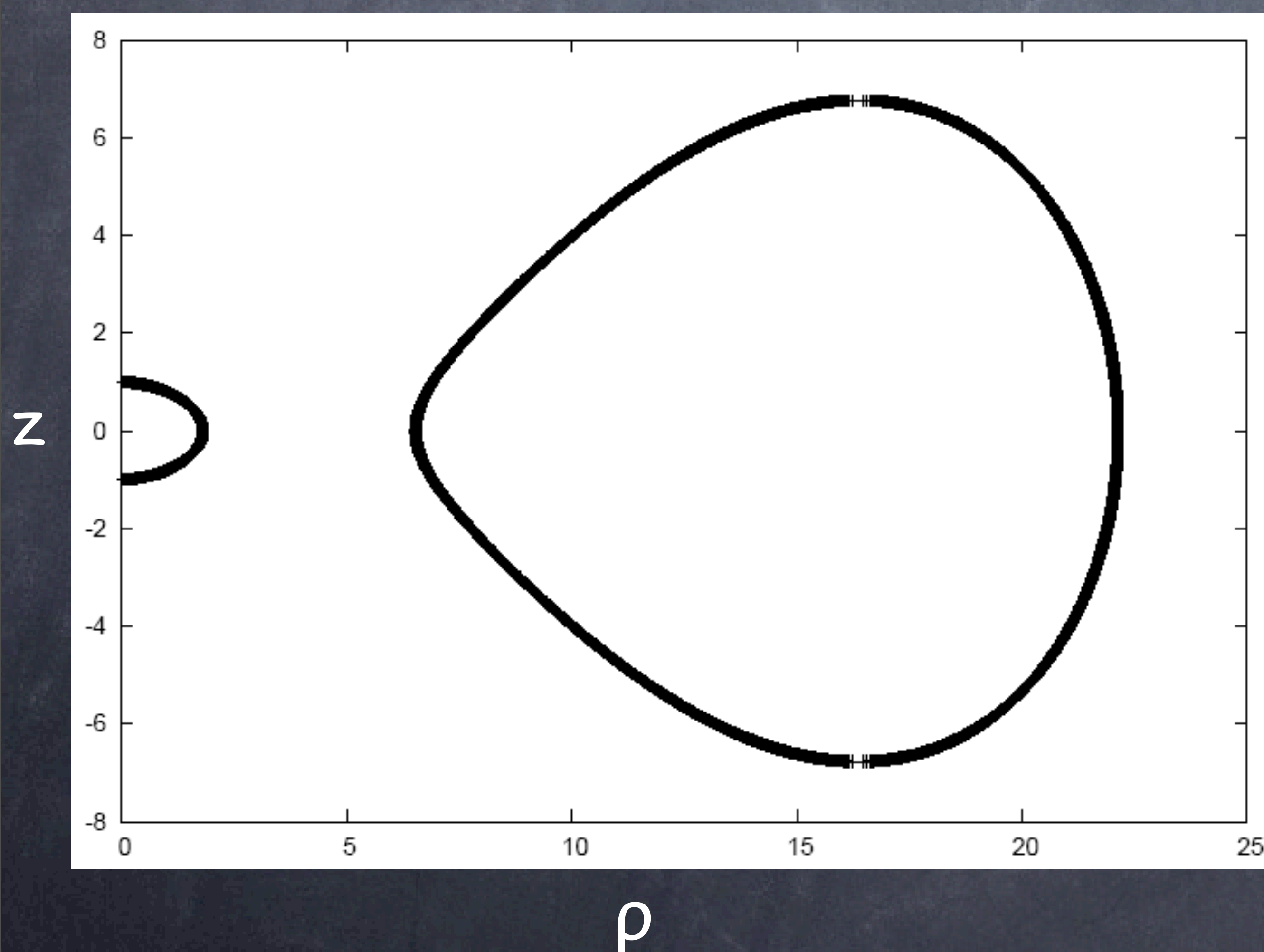


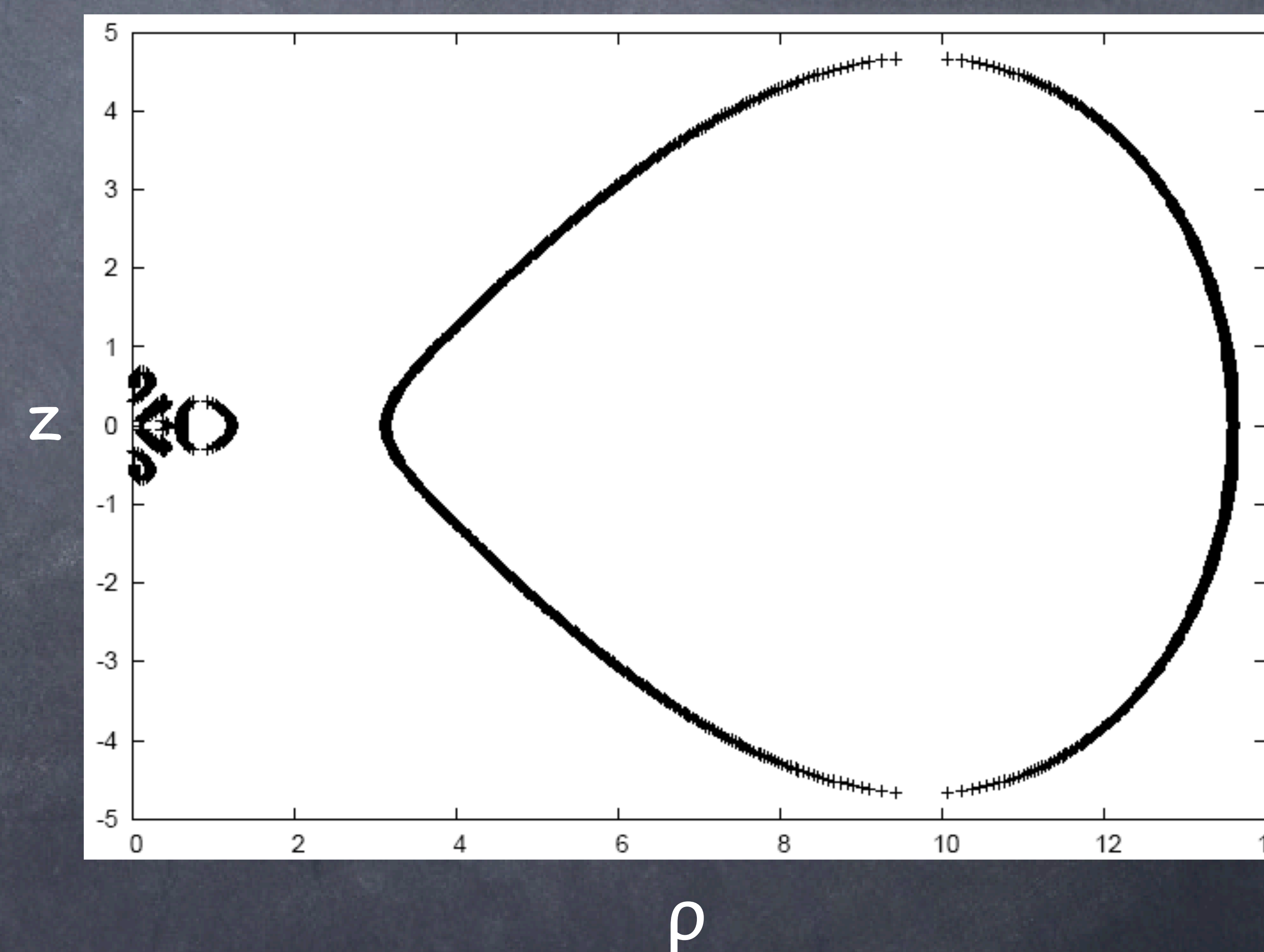


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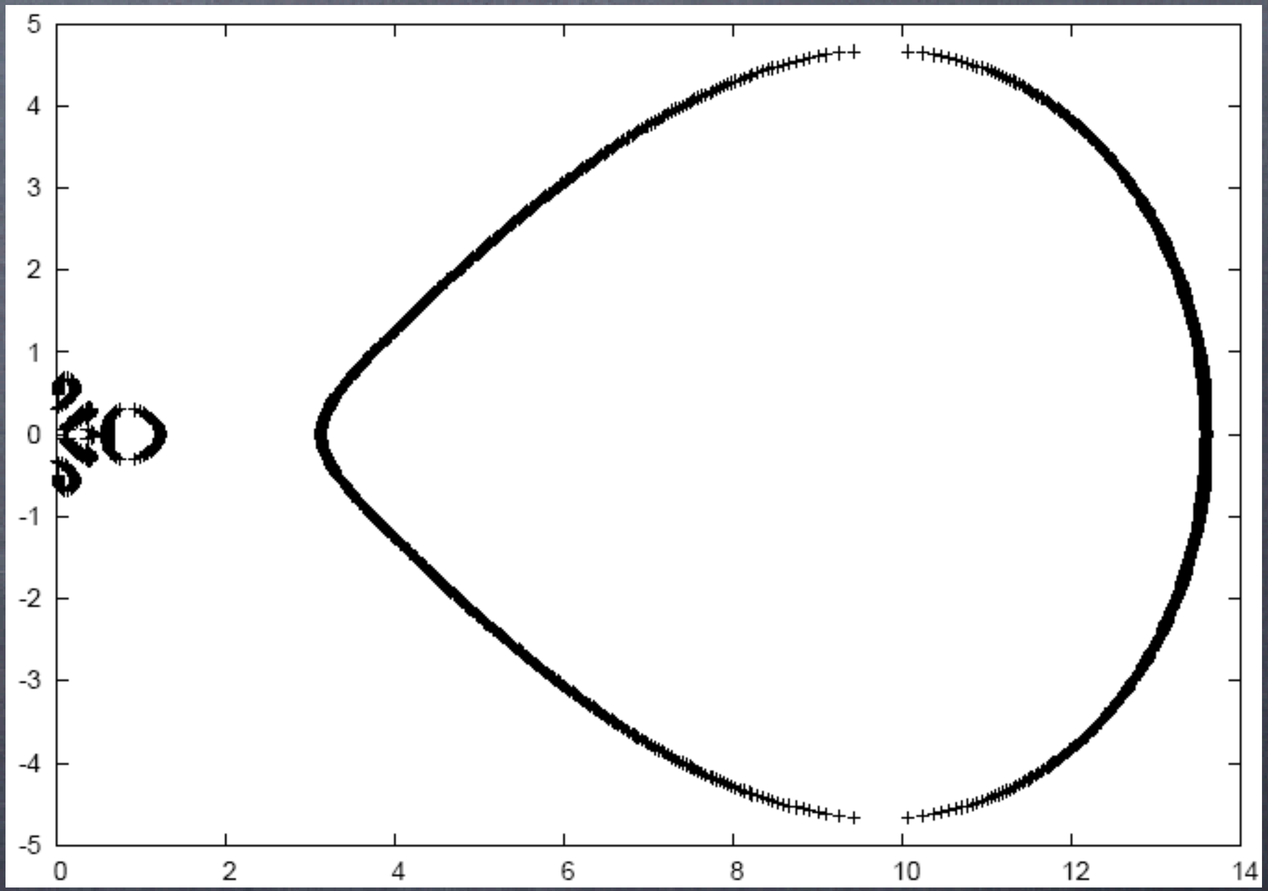
z

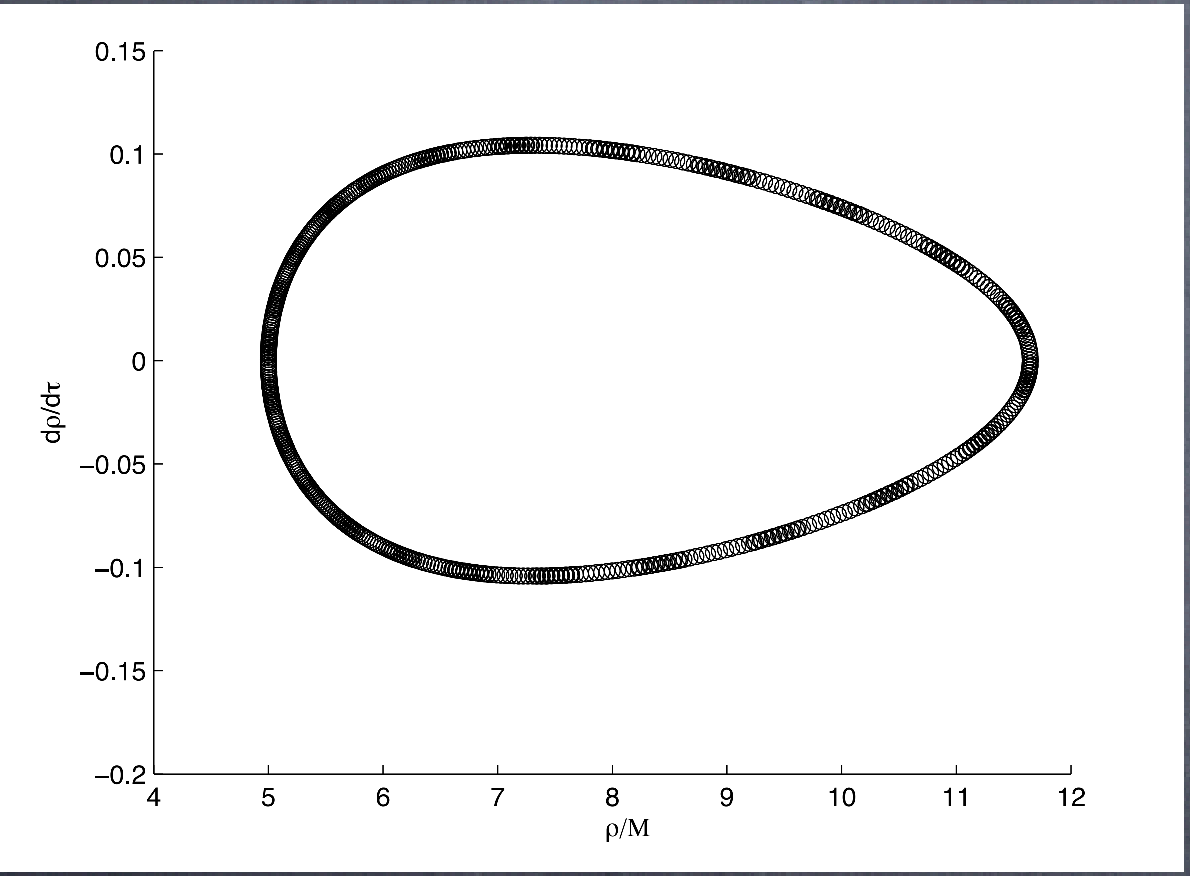


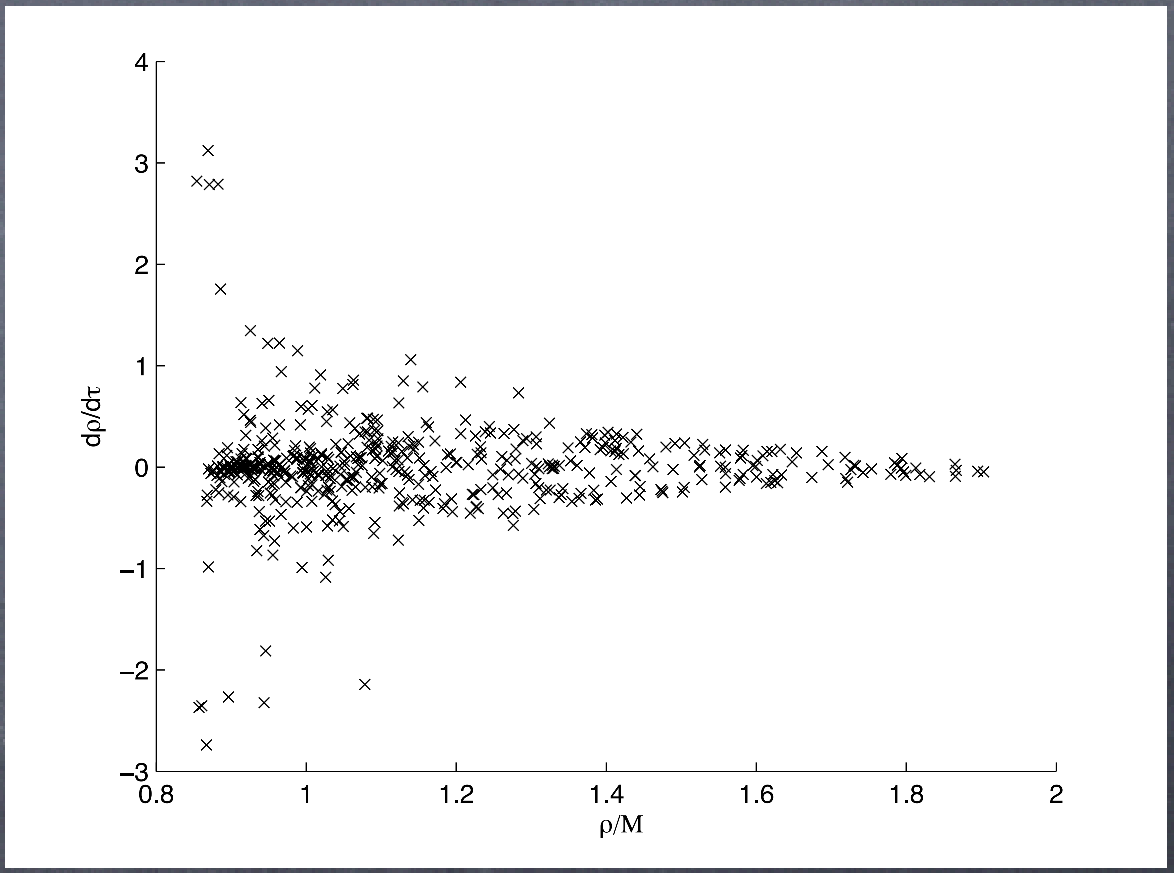

ρ

ρ

$E=0.95$, $L_z=-3$, $a/M=0.9$, $q=0.95$

# Poincare map in a bumpy spacetime, second look

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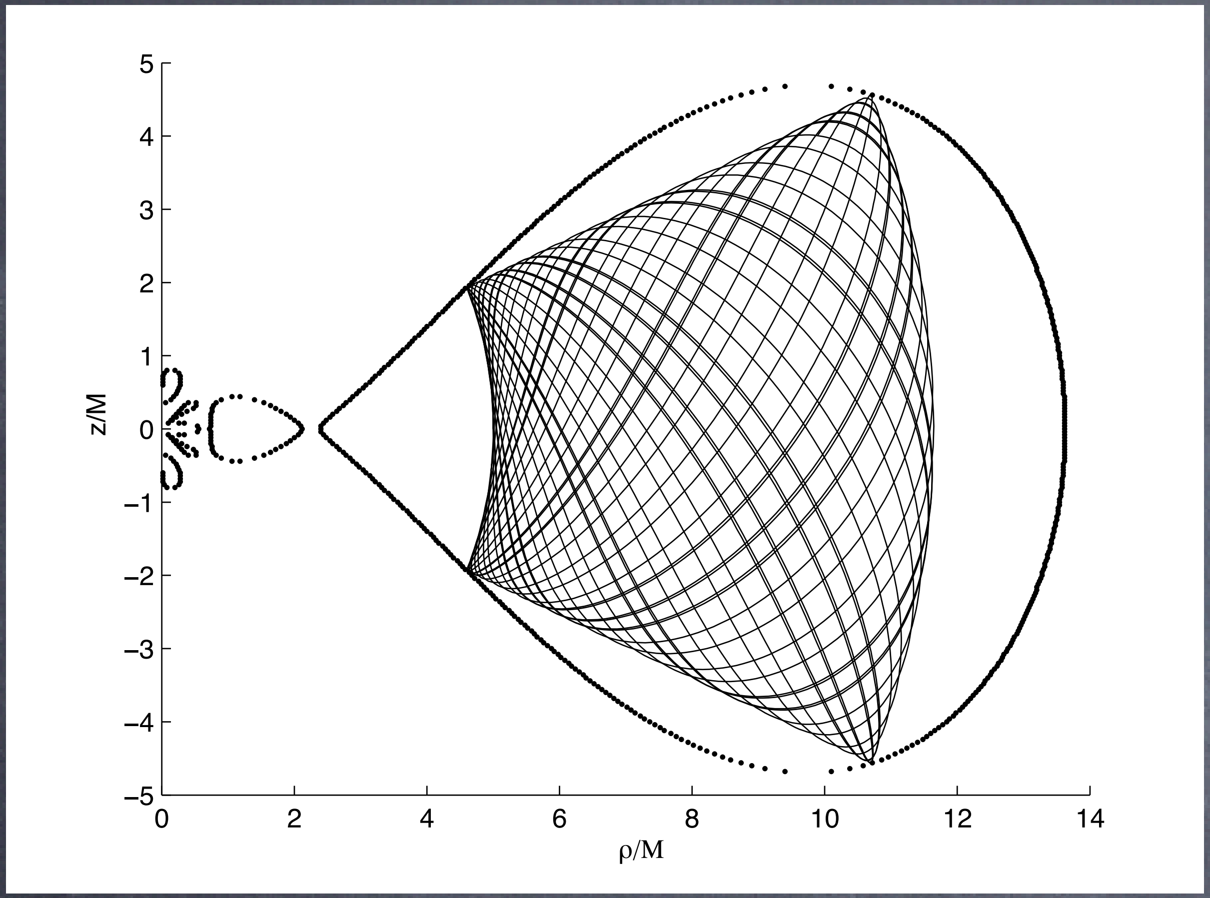
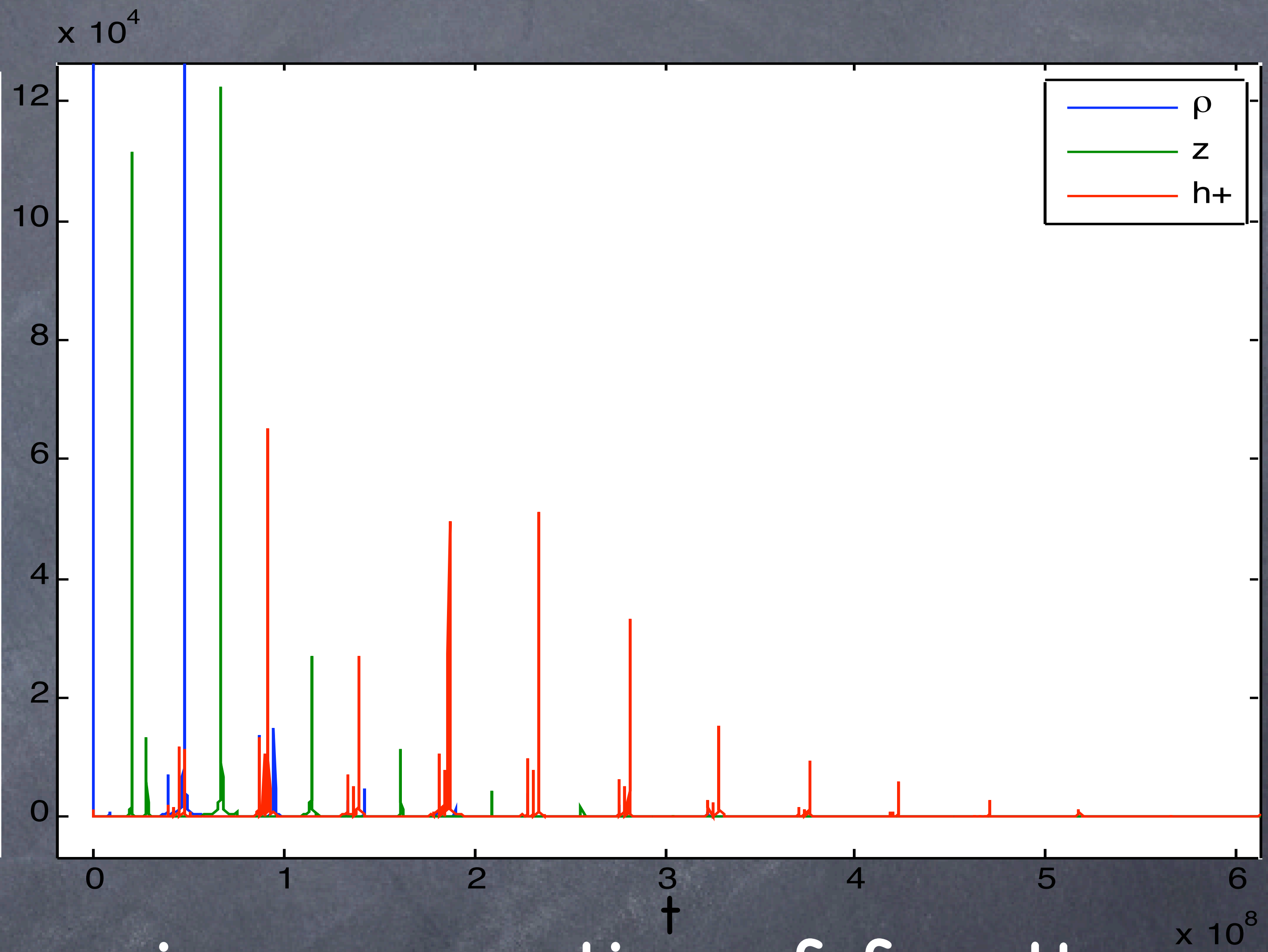
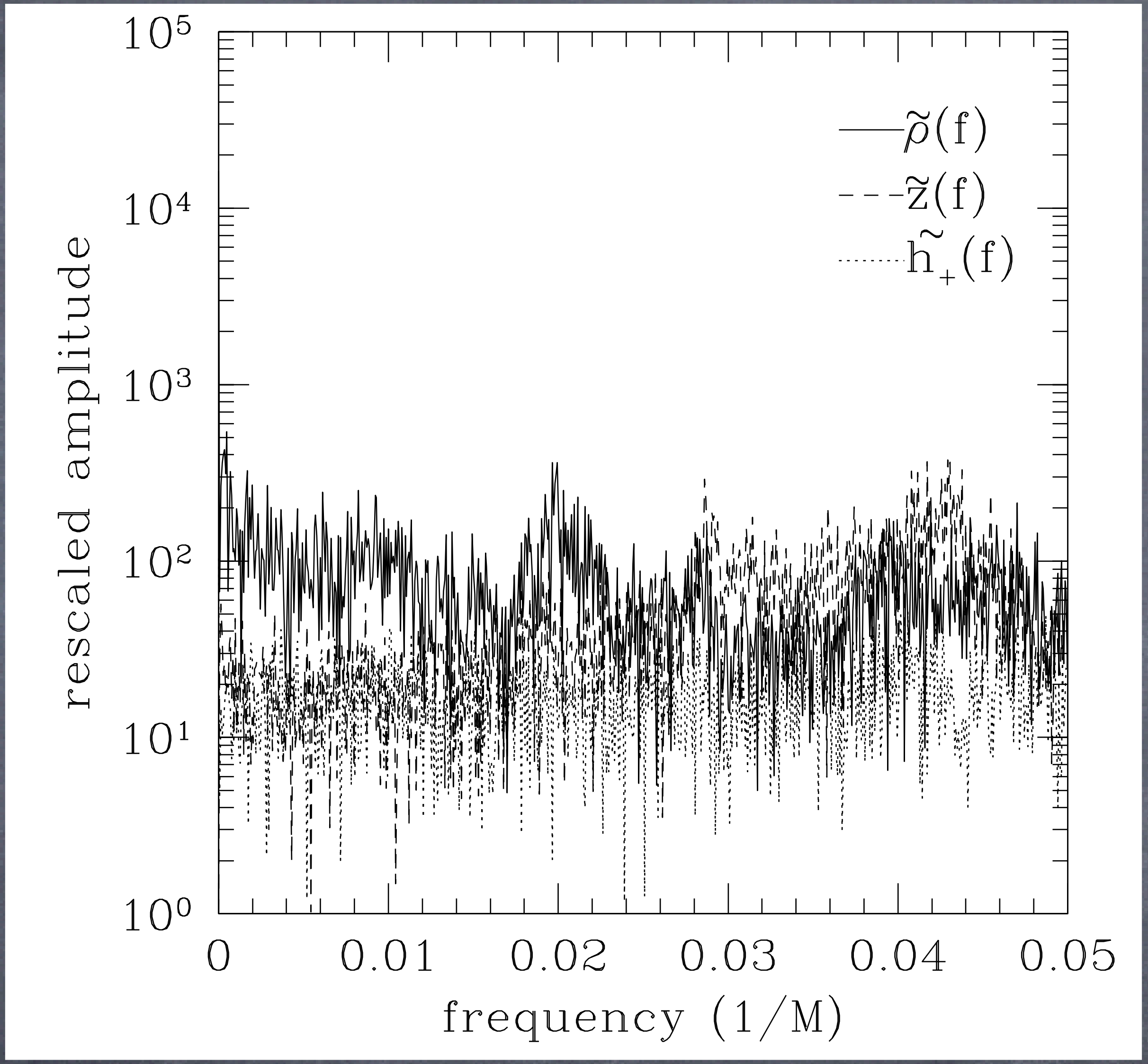
# Regular outer region



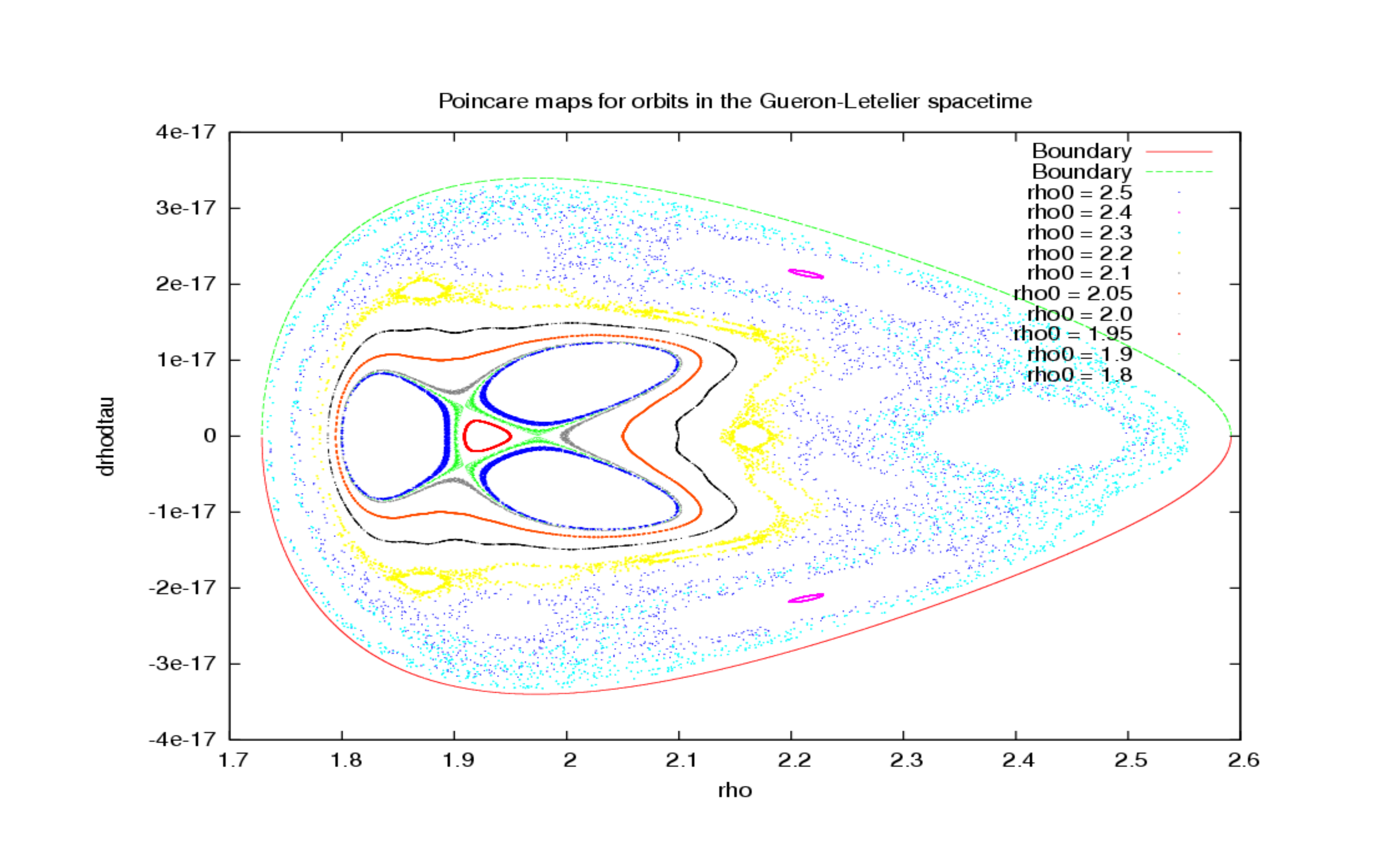

- Regular motion in outer region, suggestive of fourth-degree invariant
- Both ρ and z motion consist of harmonics of two fundamental frequencies to $10^{-7}$

# Chaotic inner region

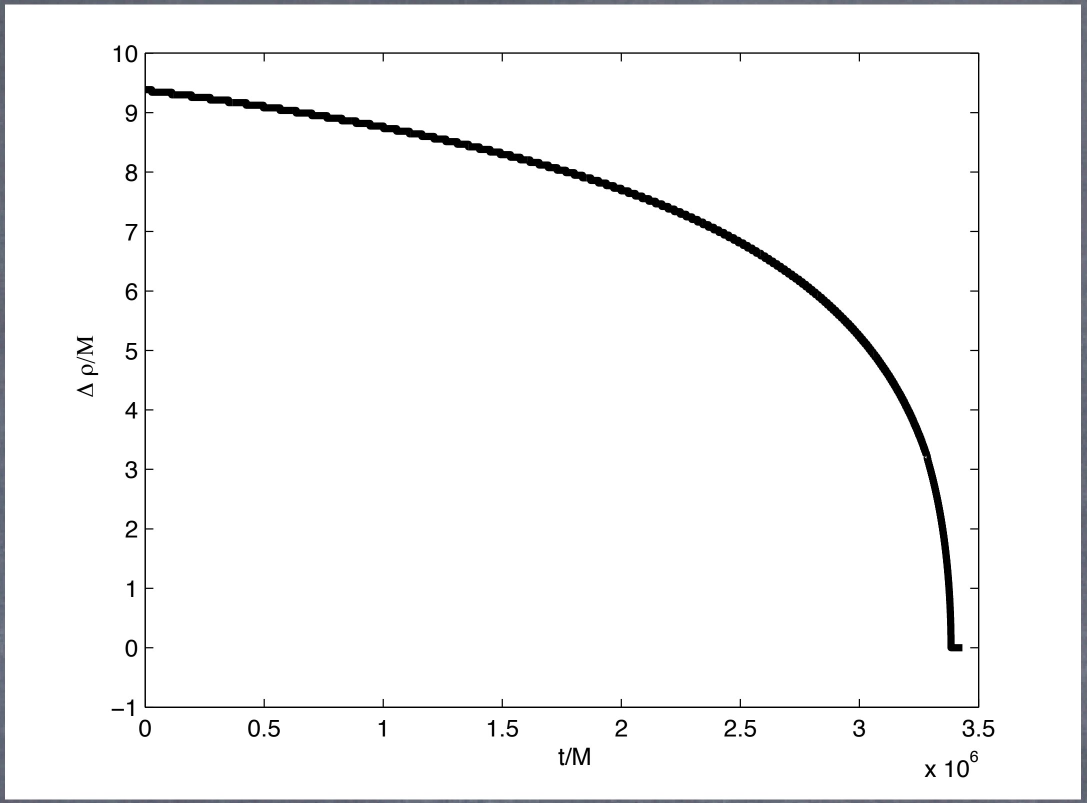



- If motion is chaotic for any initial conditions, it is chaotic for all initial conditions, but an approximate invariant may exist in some cases (invariant tori) [KAM Theorem]

# Chaos in Gueron-Letelier spacetime



# Is chaos accessible?

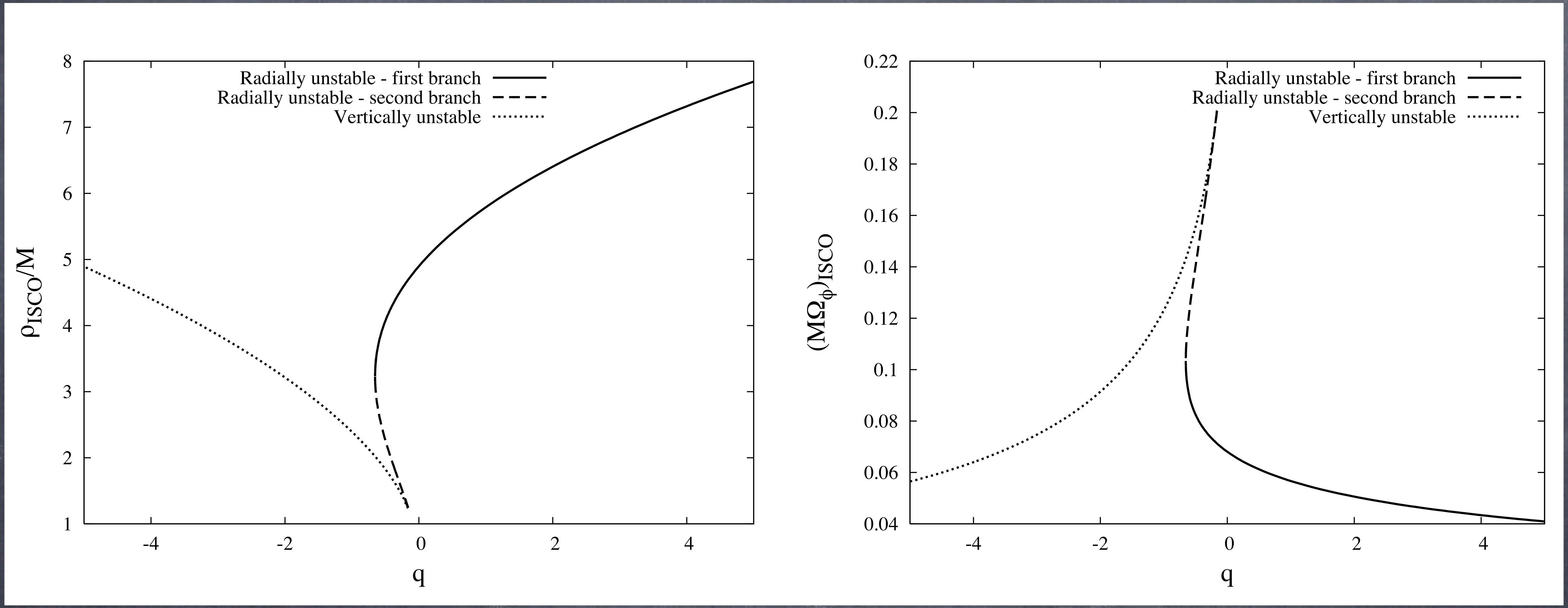



- Inner and outer regions appear to merge under radiation reaction, but never split
- Object starts out in outer, regular region; once the two regions are fully merged, motion is regular (but odd things may happen when the neck is narrow...)

# Other observable signatures of bumpiness

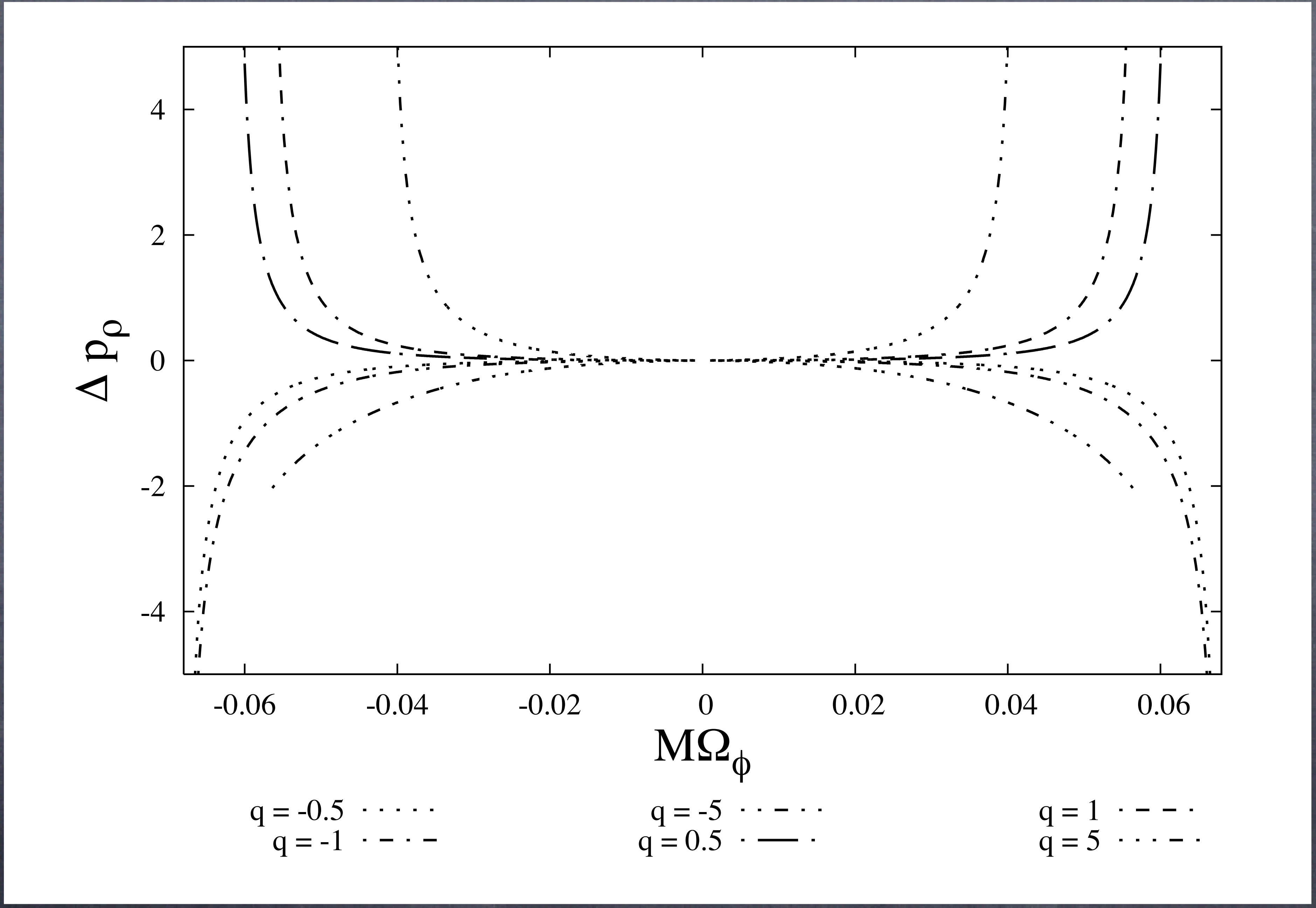
- If the orbits are indeed multi-periodic, then the spacetime “bumpiness” should be observable via:
  1. three fundamental frequencies of gravitational waves
  2. harmonic structure of the waves (relative frequencies and phases of harmonics)
  3. evolution of these with time over inspiral
- Further study required to properly analyze inspiral

# Location of innermost stable circular orbit (ISCO)

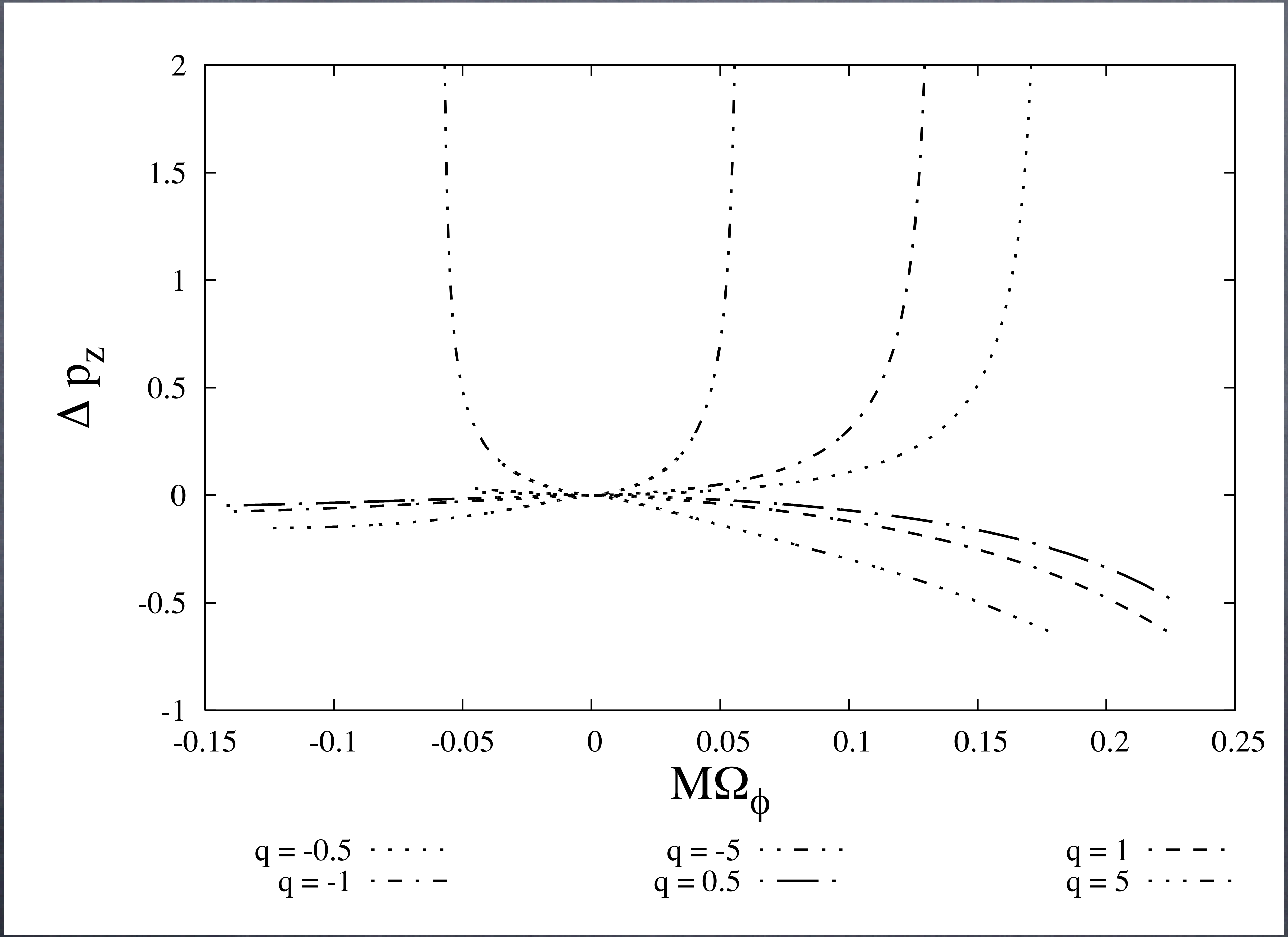



- The ISCO frequency (and hence plunge frequency) depends on the value of the spacetime quadrupole moment

# Periapsis precession



# Orbital-plane precession



# Summary

- Advanced LIGO could detect a few IMRIs per year
- Eccentricities will be low, circular waveforms can be used for detection (But should we use EMRI waveforms? Hybrid waveforms? ...?
- Gravitational waves from EMRIs should make it possible to test whether the central body [SMBH] is a Kerr black hole
- Chaos in a non-Kerr spacetime would be an obvious smoking gun, but chaotic regions are probably not accessible
- Location of ISCO, periapsis precession, and orbital-plane precession are possible observables indicating bumpiness
- Frequency evolution over inspiral would be another observable, but more work is required

# Do I really believe that IMBHs exist and MBHs are not black holes?

- I don't know. But it's dangerous to assume that one will see only what one expects to see. We should be prepared to test our assumptions.
- Every time a new part of the electromagentic spectrum was accessed (radio-astronomy, X-rays, etc.), something unexpected was seen. Gravitational waves are a new window to the universe: expect to see the unexpected!